Q10 1991 Diff Eqns

$$x\frac{dy}{dx} = \frac{1}{y} + y$$

$$\Rightarrow x\frac{dy}{dx} = \frac{1+y^2}{y}$$

CARE!

$$\Rightarrow \int \frac{y}{1+y^2}\,dx = \int \frac{dx}{x}$$

// Let $u = y^2+1$

$$\Rightarrow \frac{du}{dy} = 2y$$

$$\Rightarrow \tfrac{1}{2}du = dy$$

$$\therefore \text{LHS} \int \frac{y}{1+y^2}\,dy = \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\int \frac{du}{u}$$

$$= \tfrac{1}{2}\ln u$$

$$= \tfrac{1}{2}\ln(1+y^2) \;//$$

$$\Rightarrow \tfrac{1}{2}\ln(1+y^2) = \ln x + C$$

// $y=1$, $x=1 \Rightarrow$

$$\tfrac{1}{2}\ln(1+1^2) = \ln 1 + C$$

$$= 0 + C$$

$$\Rightarrow \tfrac{1}{2}\ln 2$$

$$\Rightarrow \ln\sqrt{2} = C \;//$$

$$\therefore \tfrac{1}{2}\ln(1+y^2) = \ln x + \ln\sqrt{2}$$

$$\Rightarrow \ln\left(\sqrt{1+y^2}\right) = \ln(x\cdot\sqrt{2})$$

$$\Rightarrow \sqrt{1+y^2} = x\sqrt{2}$$

$$\Rightarrow 1+y^2 = x^2(2)$$

$$\Rightarrow y^2 = 2x^2-1$$

$$\Rightarrow y = \pm\sqrt{2x^2-1}$$

// But $y=1$, when $x=1$,

$$\Rightarrow 1 = \pm\sqrt{2(1)^2-1} = \pm\sqrt{1}$$

$$\Rightarrow 1 = \pm 1$$

means we must select ⊕ sign //

$$\therefore \boxed{y = +\sqrt{2x^2-1}}$$

1991

(b)

Signs: + → ; ← ; $m(kv^n) = F$

$v = u$, $t = 0$, $x = 0$ —— $v = 0$, $(t = t)$, $x = s$

$$\Sigma F = ma.$$

$$\text{N II} \Rightarrow -mkv^n = m\frac{dv}{dt}.$$

$$\Rightarrow -kv^n = \frac{dv}{dt} \;\textcircled{1} \quad \text{or} \quad v\frac{dv}{dx} = -kv^n \;\textcircled{2}$$

To link v, and x select ②

$$v\frac{dv}{dx} = -kv^n.$$

$$\Rightarrow \int_u^0 \frac{v}{v^n}\,dv = \int_0^s -k\,dx.$$

$$\Rightarrow \int_u^0 v^{1-n}\,dv = -kx\Big|_0^s$$

$$\Rightarrow \frac{v^{(1-n)+1}}{(1-n)+1}\Bigg|_u^0 = -ks - (-k0)$$

$$\Rightarrow \frac{v^{2-n}}{2-n}\Bigg|_u^0 = -ks.$$

$$\Rightarrow \frac{0^{2-n}}{2-n} - \frac{u^{2-n}}{2-n} = -ks$$

$$\Rightarrow +\frac{u^{2-n}}{2-n} = ks.$$

$$\Rightarrow \boxed{s = \frac{u^{2-n}}{k(2-n)}}$$

qed.

[Easy part (b)!]